CTU
CZECH TECHNICAL UNIVERSITY IN PRAGUE

Robotics: Introduction to AI in robotics

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- Use numerical optimization to solve the minimization problem
  - dynamics (f) and costs (l, l_T) needs to be differentiable

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- What if we do not have gradient of dynamics/costs?

Reinforcement learning

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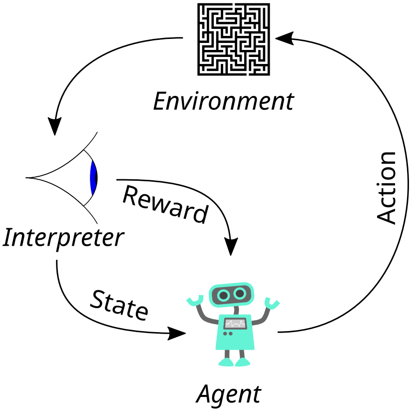

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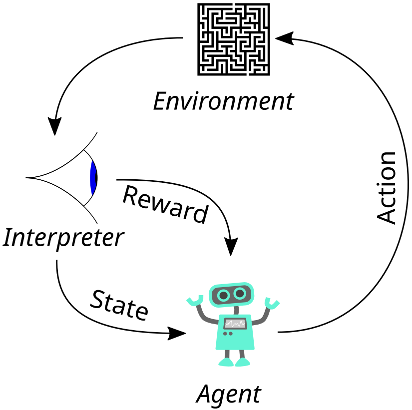

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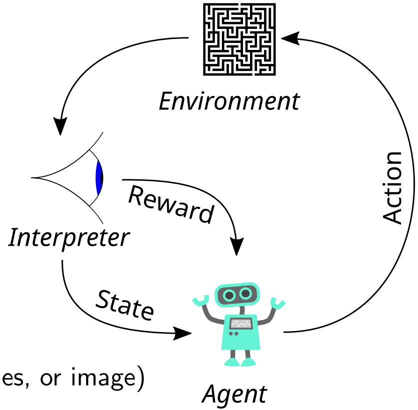

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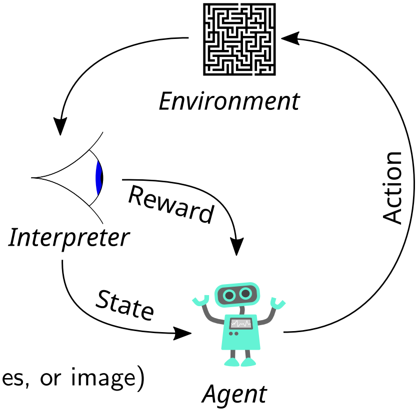

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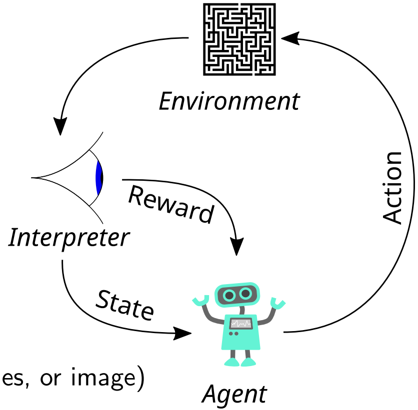

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- Goal: $\arg\max_\theta R$
- Compare to MPC: $\arg\min_{\boldsymbol{u}_1,\dots,\boldsymbol{u}_T} J$ s.t. $\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t)$

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- We need fast and accurate simulation of robots
  - Gazebo
  - NVIDIA Isaac Sim

Example of RL

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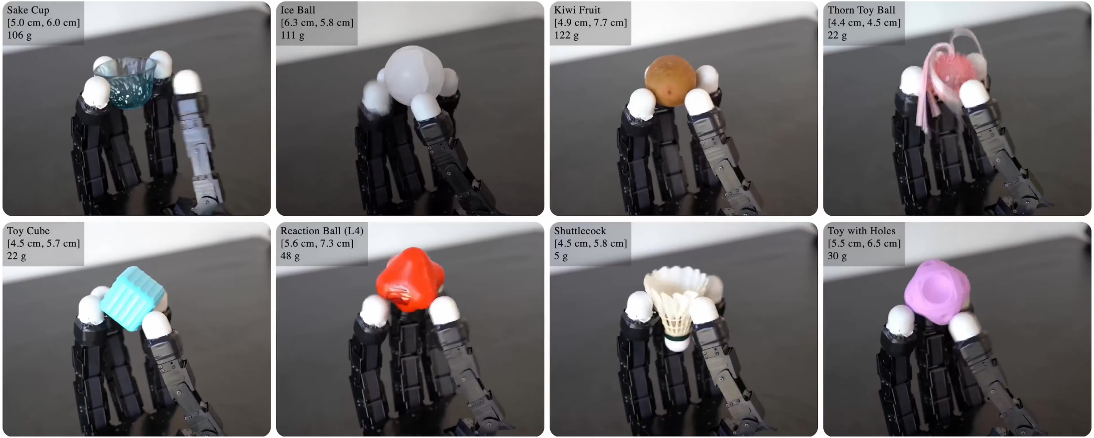

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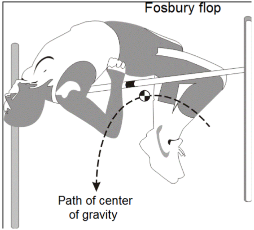
- Is there a better solution? Learning from demonstration.
- Example from high-jump (Fosbury flop - 1968 gold medal)



Offline reinforcement learning - Learning from demonstration

- Collect data from real robot guided by the operator
- Pre-Train policy on collected data
- Optionally, fine-tune policy in simulation/ on real robot
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  - behavior cloning - supervised learning
  - $\arg\min_{\theta} \sum_{i=1}^{N} (\pi_\theta(s_i) - a_i)^2$
  - diffusion policy - supervised learning

Learning from video

- Instructional videos are widely available on YouTube
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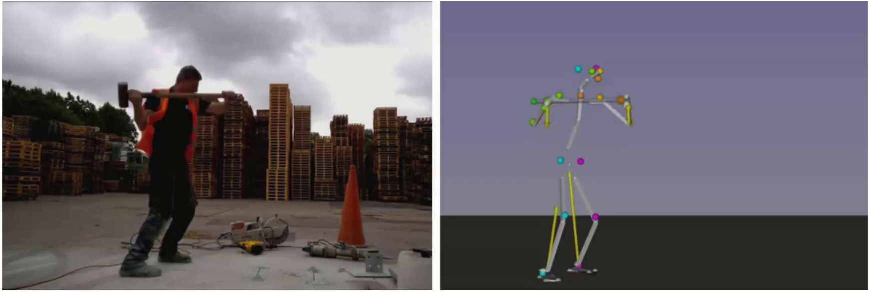
- Depends on the task/video, e.g. if human is visible
  - we can extract human pose from video
  - we can extract the manipulated object pose
  - we can extract interaction forces



Learning tool manipulation from instructional video

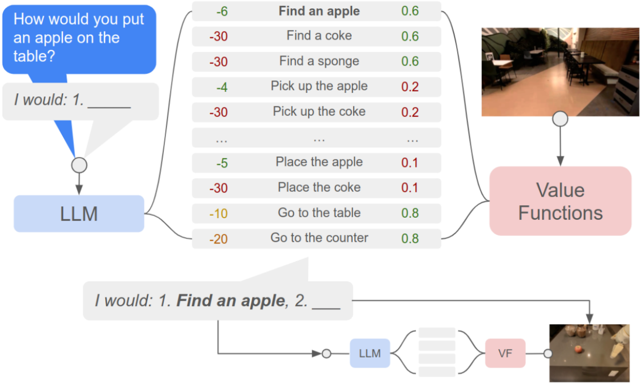
Learning to Use Tools by Watching Videos

Input: instructional video from YouTube

Output: tool manipulation skill transferred to a robot

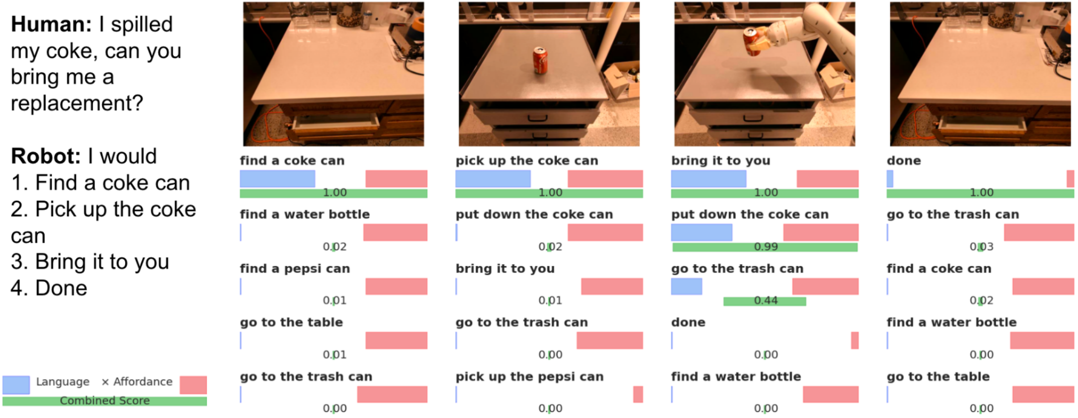
Large language models for robot learning - SayCan [1]

- Combine LLM plan with learned (RL) set of skills
  - LLM generates the global plan (prompt engineering needed)
  - Ask LLM, how much is the skill contributing to the plan
  - Ask skill, how likely it is to success



[1] https://say-can.github.io/

SayCan example



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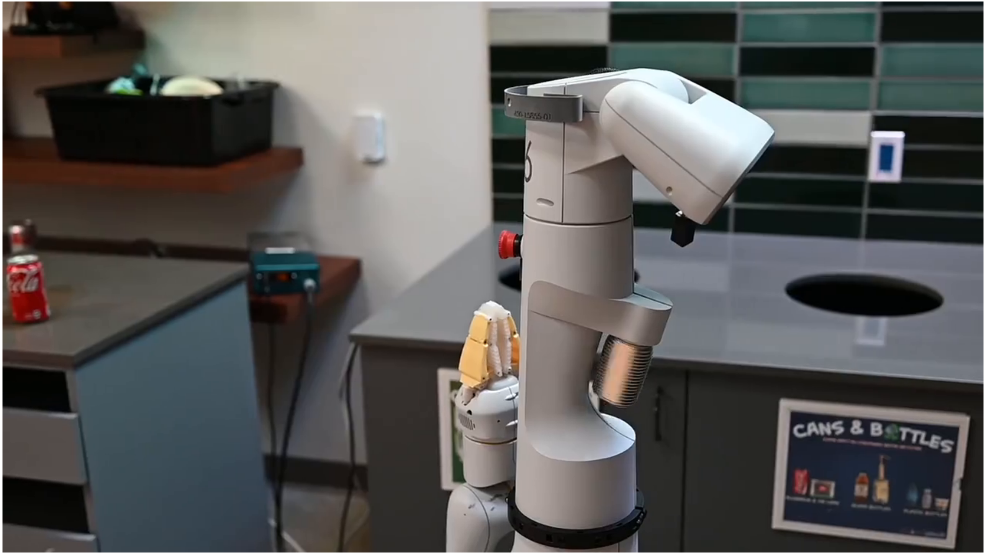

Final work

- Reservation system in BRUTE
  - reserve robot
  - reserve teacher (V. Smutný, P. Krsek, V. Petrík)
- Upload your report and code 24 hours before demonstration
- You need to demonstrate your work before signing up to exam

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- Computation of manipulator kinematics
  - forward kinematics
  - inverse kinematics
  - Jacobian