Robotika - pomocné vzorce, 13.01.2024

1 Geometrie

- Sínová věta: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
- Kosínová věta:
  - $a^2 = b^2 + c^2 - 2bc\cos\alpha$
  - $b^2 = a^2 + c^2 - 2ac\cos\beta$
  - $c^2 = a^2 + b^2 - 2ab\cos\gamma$
- Rovnice kružnice: $(x - c_x)^2 + (y - c_y)^2 = r^2$
- Rovnice přímky: $y = kx + q$
- Řešení kvadratické rovnice:
  - $ax^2 + bx + c = 0$
  - $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2 Kinematika

- Grüblerův vzorec: $n_{\mathrm{DoF}} = m\,(L-1) - \sum_{i=1}^{N} c_i = m\,(L-1-N) + \sum_{i=1}^{N} f_i$
- Rodriguesův vzorec $R(\hat{\boldsymbol{\omega}}, \theta) = I + \sin\theta\,[\hat{\boldsymbol{\omega}}] + (1-\cos\theta)\,[\hat{\boldsymbol{\omega}}]^2$
- Skew-symmetric matice $[\boldsymbol{\omega}] = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$
- Algoritmus osa-úhel z R
  - Pokud $R = I$ pak $\theta = 0$ a $\hat{\boldsymbol{\omega}}$ nedefinována.
  - Pokud $\operatorname{tr} R = -1$ pak $\theta = \pi$ a
    - $\hat{\boldsymbol{\omega}} = \frac{1}{\sqrt{2(1+r_{33})}} \begin{pmatrix} r_{13} & r_{23} & 1 + r_{33} \end{pmatrix}^\top$ pokud $r_{33} \neq -1$
    - $\hat{\boldsymbol{\omega}} = \frac{1}{\sqrt{2(1+r_{22})}} \begin{pmatrix} r_{12} & 1 + r_{22} & r_{32} \end{pmatrix}^\top$ pokud $r_{22} \neq -1$
    - $\hat{\boldsymbol{\omega}} = \frac{1}{\sqrt{2(1+r_{11})}} \begin{pmatrix} 1 + r_{11} & r_{21} & r_{31} \end{pmatrix}^\top$ pokud $r_{11} \neq -1$
  - Jinak $\theta = \arccos\left(1/2\,(\operatorname{tr} R - 1)\right)$ a $[\hat{\boldsymbol{\omega}}] = \frac{1}{2\sin\theta}(R - R^\top)$
- Quaternion z R
  - $q_w = 1/2\sqrt{1 + \operatorname{tr} R}$
  - $\boldsymbol{q}_{xyz} = \frac{1}{4q_w} \begin{pmatrix} r_{32} - r_{23} & r_{13} - r_{31} & r_{21} - r_{12} \end{pmatrix}^\top$

3 Dynamika

- Lagrangián
  $\mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \mathcal{K}(\boldsymbol{q}, \dot{\boldsymbol{q}}) - \mathcal{P}(\boldsymbol{q})$
  - Kinetic energy $\mathcal{K}(\boldsymbol{q}, \dot{\boldsymbol{q}})$
  - Potential energy $\mathcal{P}(\boldsymbol{q})$
- Rovnice pohybu: $\boldsymbol{\tau} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}}} - \frac{\partial \mathcal{L}}{\partial \boldsymbol{q}}$
- Pocitová hmotnost chapadla:
  - Kinetic energy must be constant: $\frac{1}{2}V^\top \Lambda(\boldsymbol{q}) V = \frac{1}{2}\dot{\boldsymbol{q}}^\top M(\boldsymbol{q})\dot{\boldsymbol{q}}$
    - $\Lambda(\boldsymbol{q})$ effective mass of end-effector
    - $V = \left(\dot{x}, \dot{y}\right)^\top$ velocity of end-effector
  - Jacobian $V = J(\boldsymbol{q})\dot{\boldsymbol{q}}$
  - $V^\top \Lambda(\boldsymbol{q}) V = (J^{-1}V)^\top M(\boldsymbol{q})(J^{-1}V)^\top = V^\top (J^{-\top} M(\boldsymbol{q}) J^{-1}) V$
  - End-effector mass matrix: $\Lambda(\boldsymbol{q}) = J^{-\top}(\boldsymbol{q}) M(\boldsymbol{q}) J^{-1}(\boldsymbol{q})$
- Dynamika s omezením
  - Robot omezen pomocí k rychlostních omezení
    - $A(\boldsymbol{q})\dot{\boldsymbol{q}} = 0, A \in \mathbb{R}^{k \times n}$
  - Rovnice pohybu
    - $\boldsymbol{\tau} = M(\boldsymbol{q})\ddot{\boldsymbol{q}} + h(\boldsymbol{q}, \dot{\boldsymbol{q}}) + A^\top(\boldsymbol{q})\boldsymbol{\lambda}, \quad \text{s.t. } A(\boldsymbol{q})\dot{\boldsymbol{q}} = 0$
    - $\boldsymbol{\lambda}$ - vektor Lagrangeových multiplikátorů
    - $A^\top(\boldsymbol{q})\boldsymbol{\lambda}$ - síla působící proti omezením vyjádřená jako kloubové síly/ momenty
    - Lambdu lze vypočítat analyticky:
      $\boldsymbol{\lambda} = (AM^{-1}A^\top)^{-1}(AM^{-1}(\boldsymbol{\tau} - h) + \dot{A}\dot{\boldsymbol{q}})$