Robotics: Closed kinematics chains

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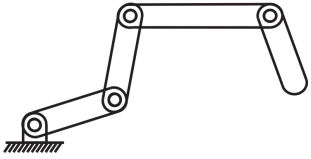
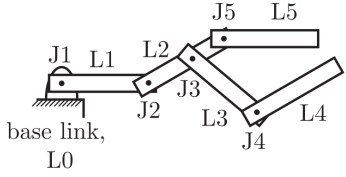
Open/Closed kinematic chain

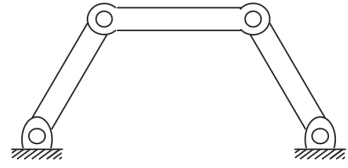
- Open kinematics chains: no loops
- Closed kinematic chains contains loops
- Many closed kinematics chains can be expressed with open kinematics chains



Open - sequential



Open - tree



Closed

Examples of closed kinematics chain

Closed kinematics chains

- Closed kinematics chains
  - Contain loops
  - Are more difficult to control
  - Typically small workspace
  - Are more difficult to analyze
- Advantages
  - Can be redundantly actuated
  - Mechanical advantage: faster, stronger, or stiffer
- Grübler's formula can be used to determine number of DoF

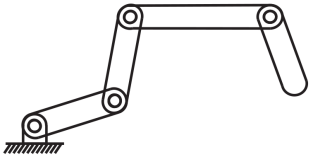
Grübler's formula

- $n_{\text{DoF}} = m\,(L-1) - \sum_{i=1}^{N} c_i = m\,(L-1-N) + \sum_{i=1}^{N} f_i$
- L is number of links including ground
- N is number joints
- m is DoF of rigid body (3 for planar, 6 for spatial)
- c_i number of constrains provided by joint i
- f_i number of freedoms provided by joint i
- $f_i + c_i = m$
- Works for *generic* cases, fails under certain configurations - when joints constrains are not independent

Applications of Grübler's formula

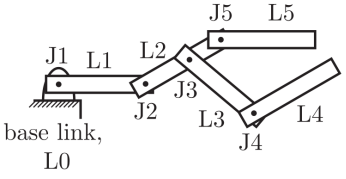
$n_{\text{DoF}} = m\,(L - 1 - N) + \sum_{i=1}^{N} f_i$

m - body DoF, L - number of links, N - number of joints, f_i - joint DoF

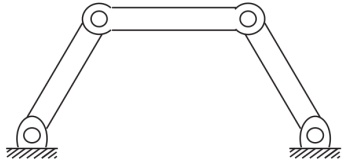


$3(5-1-4)+(1+1+1+1) = 4$ DoF

$3(6-1-5)+(1+1+1+1+0) = 4$ DoF (if fixed joint)



$3(6-1-5)+(1+1+1+1+1) = 5$ DoF



$3(4-1-4)+(1+1+1+1) = 1$ DoF

$3(5-1-5)+(1+1+1+1+0) = 1$ DoF (if two grounds connected by fixed joint)

Examples of kinematics chains

- Open kinematics chains
  - Structure RR, Task space $\mathbb{R}^2$
  - Structure RRR, Task space $SE(2)$
- Closed kinematics chains
  - Structure RRRR, Task space $\mathbb{R}^1$
  - Structure RRRRRR, Task space $SE(2)$
  - Structure RRRP, Task space $\mathbb{R}^1$

Spatial inverse kinematics for Mitsubishi RV6S