CTU

CZECH TECHNICAL UNIVERSITY IN PRAGUE

Robotics: Introduction to perception

Vladimír Petrík

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What is image?

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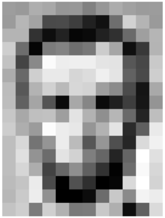
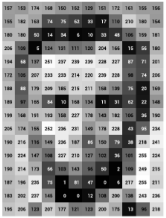
- Image is array of numbers[1]





157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
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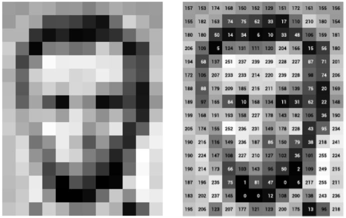

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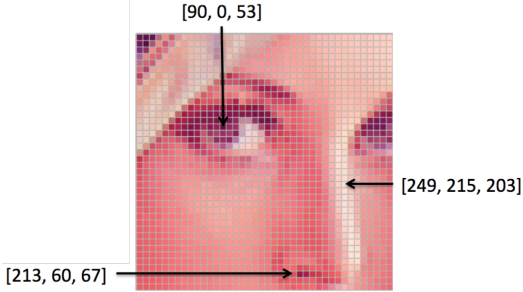

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- Perspective camera
  - pinhole camera model[2]
  - projects spatial point $\boldsymbol{x}_c$ into image point $\boldsymbol{u} = \begin{pmatrix} u & v \end{pmatrix}^\top$ by intersecting
    - image plane and
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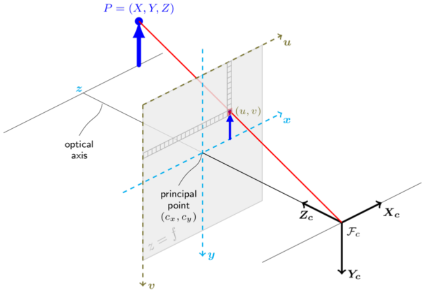

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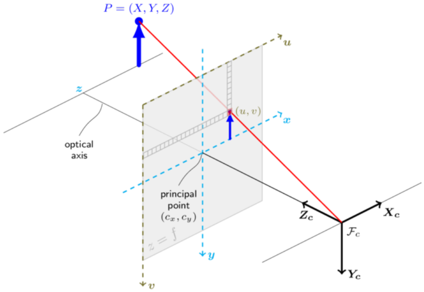

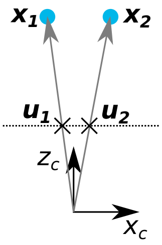

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Projection of pinhole camera

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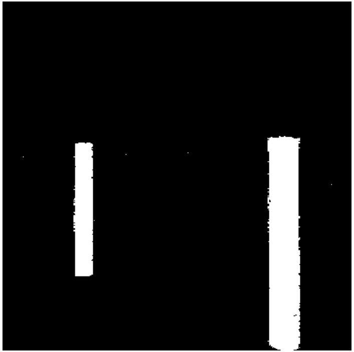

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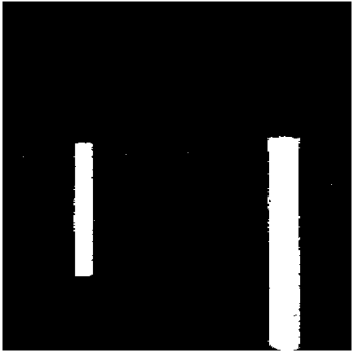

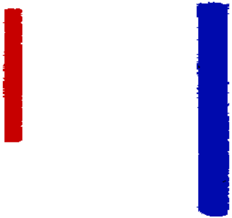

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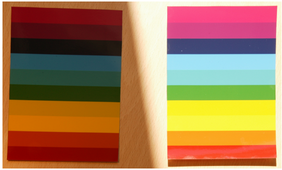

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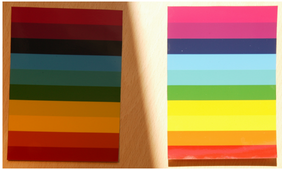

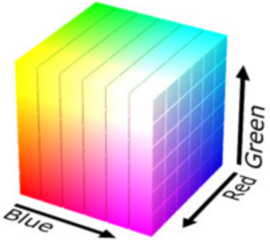

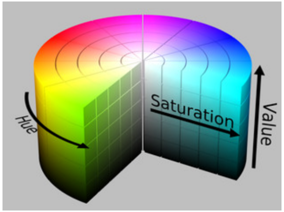

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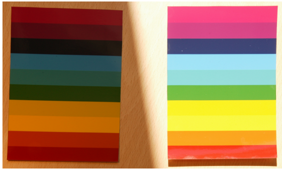
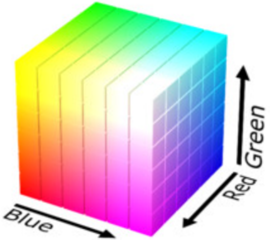
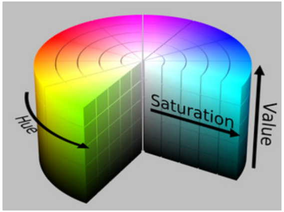
  - compute connected components
  - remove small or deformed segments
  - assign label based on thresholds







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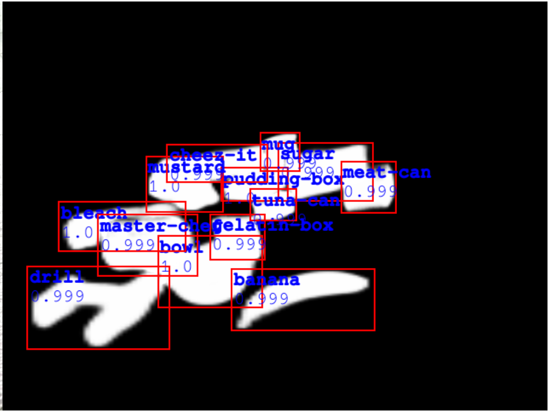
- Inference:
  - Input: image
  - Output: segmentation mask, bounding box, label, and confidence

Mask R-CNN results



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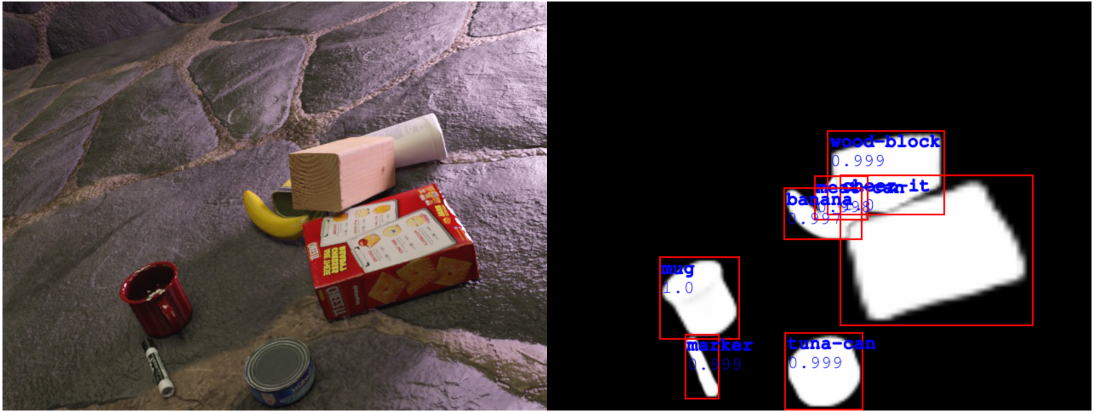

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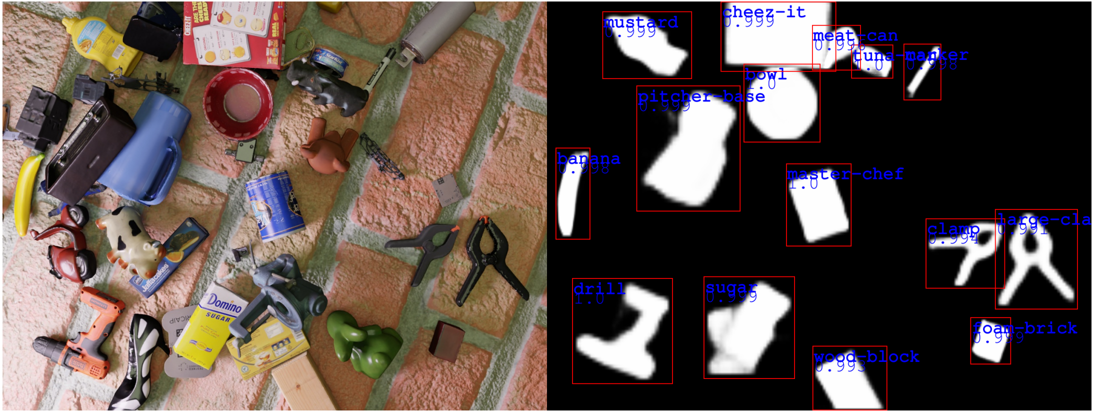

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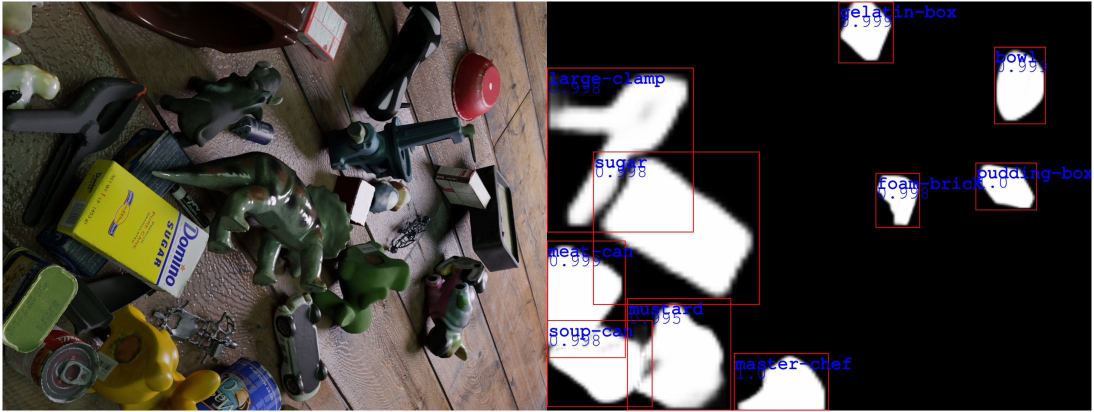

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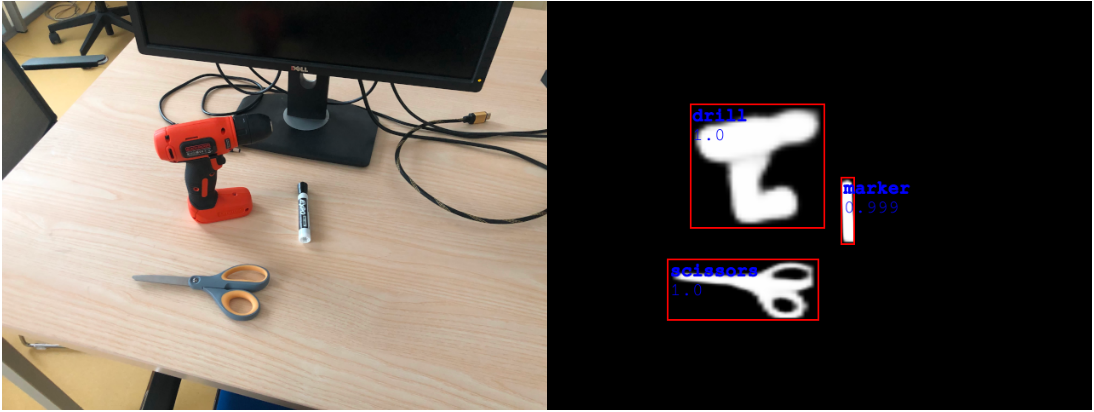

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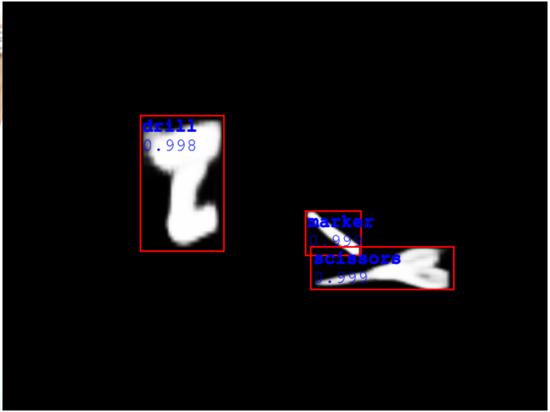
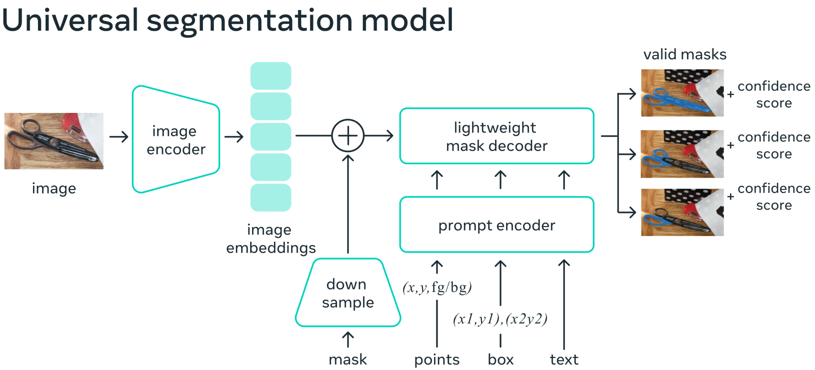


Segmentation masks without re-training

- Segment Anything Model (SAM)
  - segment any object, in any image, with a single click
  - dataset of 10M images, 1B masks



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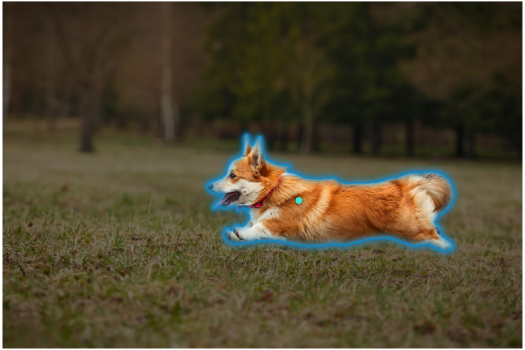

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SAMv2

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- Information only in image space
- How to use it in robot space?

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  - analyzed by **homography**

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    - position of cube center w.r.t. table corner
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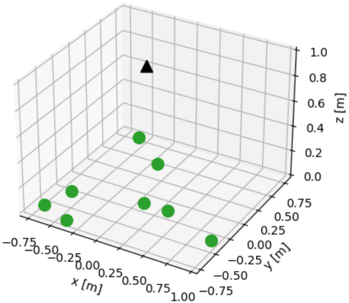
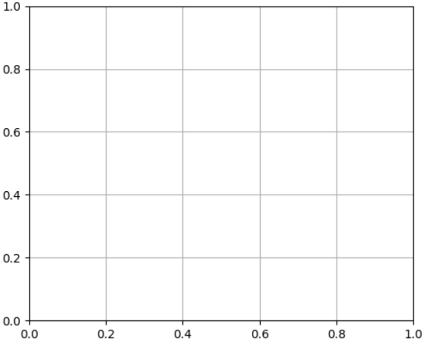
Homography example





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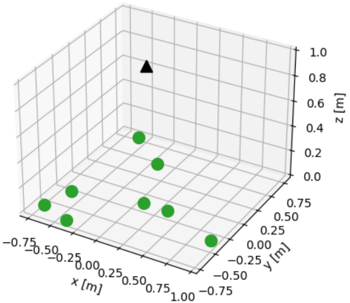

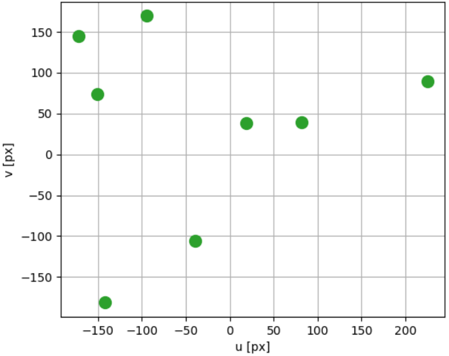

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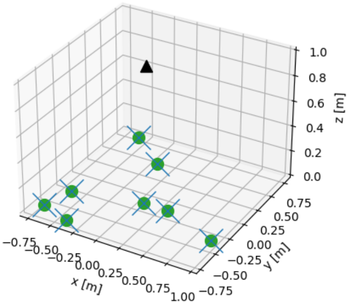

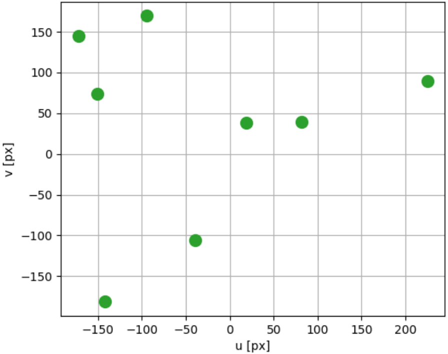

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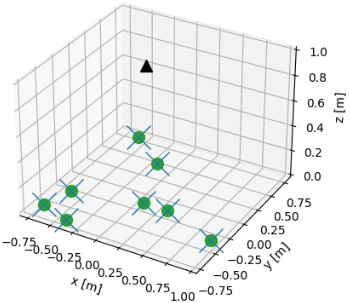

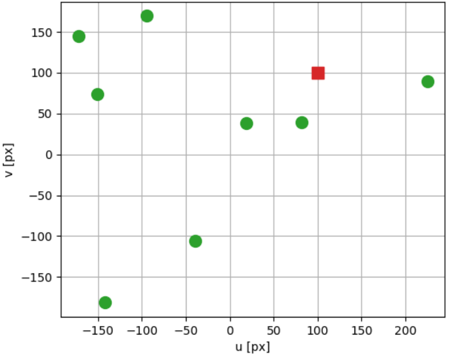

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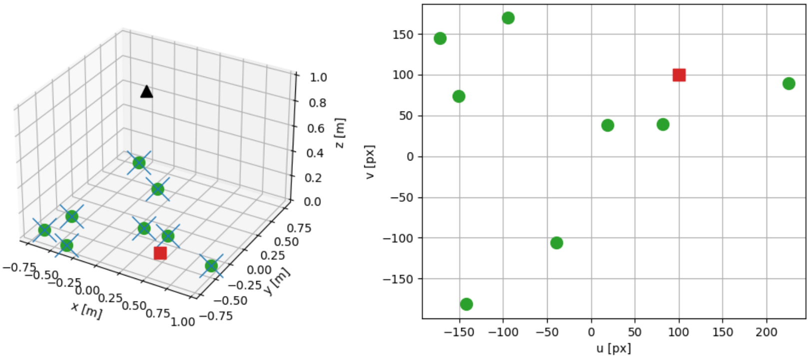
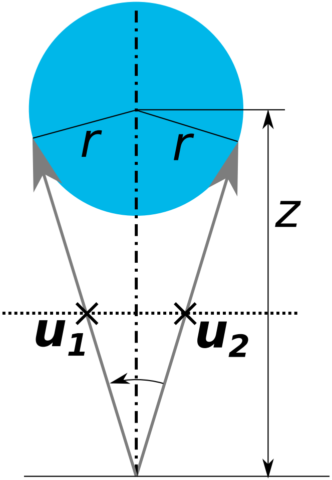


Non-planar pose estimation

- Homography maps only plane to plane
- More general object pose estimation in **camera** frame
  - get depth by mapping from area in pixels to depth for fixed size objects
  - get depth by additional scene information, *e.g.* known size/model of the objects
  - RGBD camera
  - additional markers

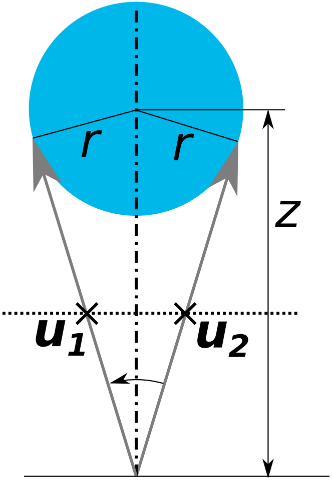
Using prior knowledge about size

- We know radius is fixed



Using prior knowledge about size

- We know radius is fixed
- From detected pixels $\boldsymbol{u}_1, \boldsymbol{u}_2$, we can compute rays $\boldsymbol{x}_1, \boldsymbol{x}_2$:
  $\frac{1}{\lambda_i}\boldsymbol{x}_i = K^{-1}\boldsymbol{u}_i$



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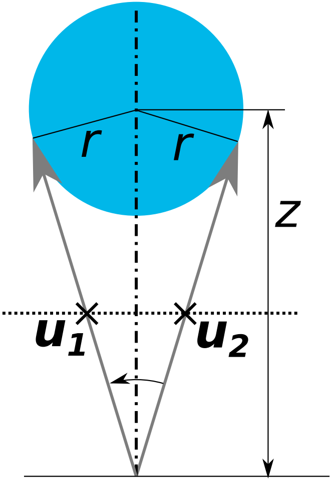
- Angle between vectors: $\cos\alpha = \frac{\frac{1}{\lambda_1\lambda_2}}{\frac{1}{\lambda_1\lambda_2}} \frac{\boldsymbol{x}_1 \cdot \boldsymbol{x}_2}{\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\|}$



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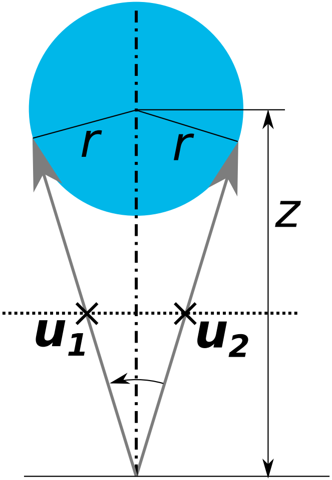
- Angle between vectors: $\cos\alpha = \frac{\frac{1}{\lambda_1\lambda_2}}{\frac{1}{\lambda_1\lambda_2}} \frac{\boldsymbol{x}_1 \cdot \boldsymbol{x}_2}{\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\|}$
- Depth: $z = \frac{r}{\sin(\alpha/2)}$

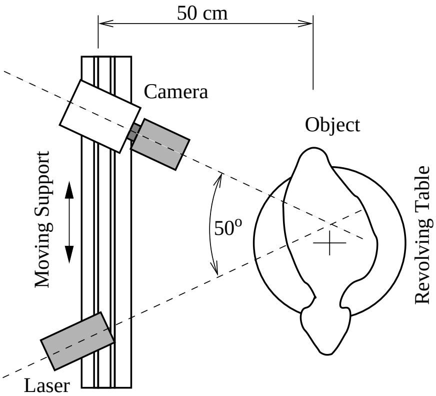


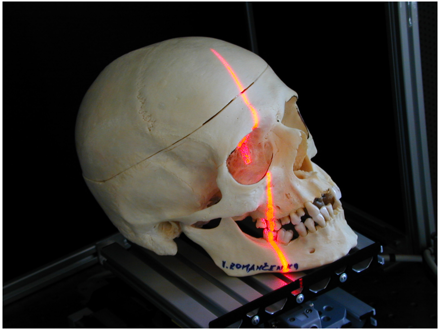
Using depth sensor

- RGB**D** sensors
  - RGB image $(H \times W \times 3)$
  - Depth map $(H \times W \times 1)$, distance in meters for each pixel
  - Structured point cloud $(H \times W \times 3)$, $\begin{pmatrix} x_c & y_c & z_c \end{pmatrix}$ for each pixel

How depth sensor works

- Laser projects pattern and camera recognizes it
- Depth information is computed using triangulation



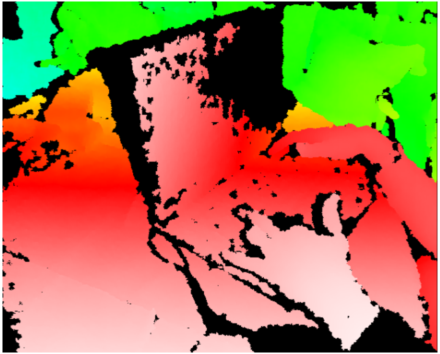


2D depth sensors

- Based on the structured light
- Projects 2D infra red patterns
- One projector and two cameras (RGB + IR)

Issues with depth sensors

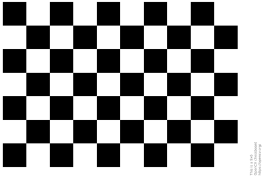
- Depth reconstruction is not perfect (black areas in the image[3])
- In python represented by NaN
- Not every pixel in RGB has reconstructed depth value
- RGB and Depth data are not aligned (you need to calibrate them)

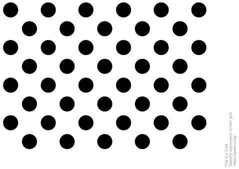


[3] https://commons.wikimedia.org, User:Kolossos

Additional markers

- Can we compute the pose of patterns[4]?

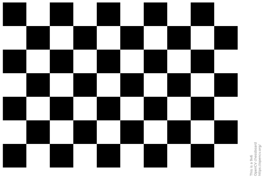


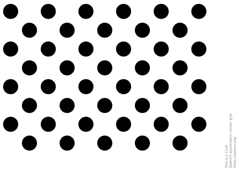


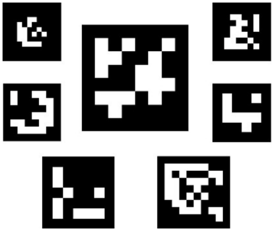
[4] docs.opencv.org

Additional markers

- Can we compute the pose of patterns[4]?
  - the size and structure needs to be known
  - subpixel accuracy
  - it has to be completely visible
- Can we compute the pose of ArUco markers?



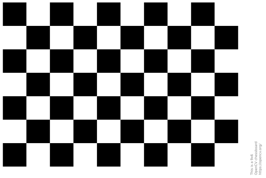


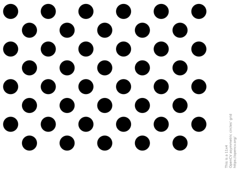


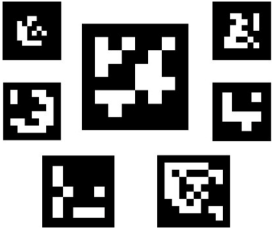
[4] `docs.opencv.org`

Additional markers

- Can we compute the pose of patterns[4]?
  - the size and structure needs to be known
  - subpixel accuracy
  - it has to be completely visible
- Can we compute the pose of ArUco markers?
  - less accurate than regular patterns
  - provides marker id and the pose
  - it has to be completely visible

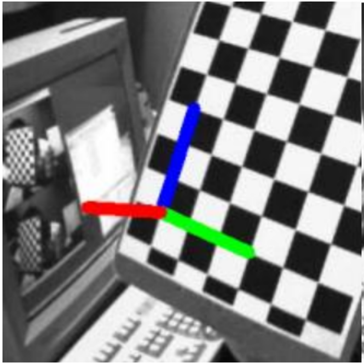
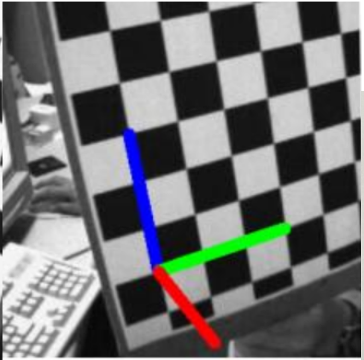
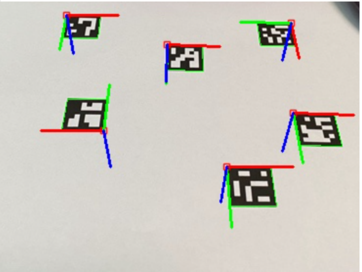






[4] docs.opencv.org

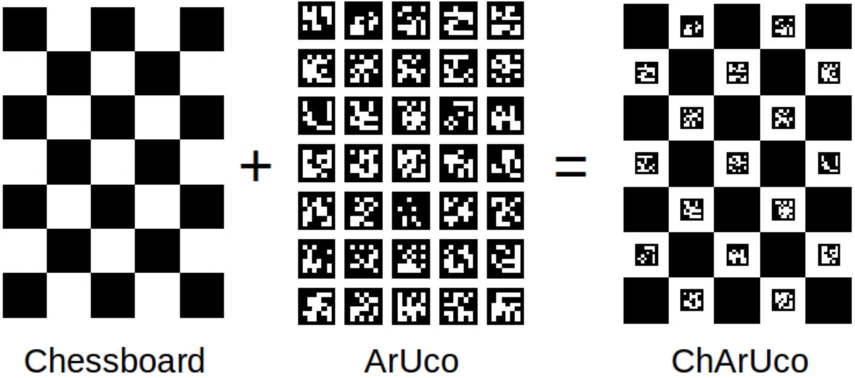
Markers pose example

Markers example from real-world

ChArUco board for calibration

- Combines accuracy of pattern with detections of ArUco
- Partial visibility detections



Charuco definition

Camera matrix estimation with boards

- We can estimate camera matrix from correspondences in image space and spatial space
  - collect images of the board from different views
  - detect boards
  - compute correspondences between image points and board frame points
  - ```
    _, K, dist_coeffs, rvecs, tvecs = cv2.calibrateCamera(
        obj_points, img_points, img_shape)
    ```
- In addition we get
  - distortion coefficients that compensates defects of objective
    ```
    Knew, roi = cv.getOptimalNewCameraMatrix(K, dist_coeffs,
        img_shape, 1, img_shape)
    img_undistorted = cv.undistort(img, K, dist_coeffs, None, Knew)
    ```
  - $SE(3)$ poses of boards in camera frame
    ```

Pose estimation from RGB(D)

- Pose estimation methods
  - use prior knowledge about the task, *e.g.* fixed height objects on a plane
  - use prior knowledge about the objects (size)
  - use depth sensor
  - use ArUco markers

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  - homography estimates poses of objects w.r.t. plane frame

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  - use ArUco markers
- Where is robot?
  - homography estimates poses of objects w.r.t. plane frame
  - other methods estimate poses in camera frame
  - we need to estimate/calibrate T_{RC}

Summary

- Image representation
- Projection to/from image
- Segmentation in image space
- Homography
- Pose estimation from image
- Camera calibration

Laboratory

- No new homework this week
- Mandatory excercise - Safety
  - CIIRC, B670

What is next?

- Next week, we will start with a test
  - Questions from the first two lectures
  - SE2 and SE3 transformations and properties
  - Forward kinematics a DoF