Robotics: Calibration

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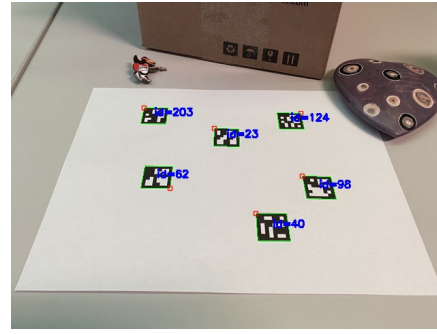
  - Checkerboard
  - Aruco markers



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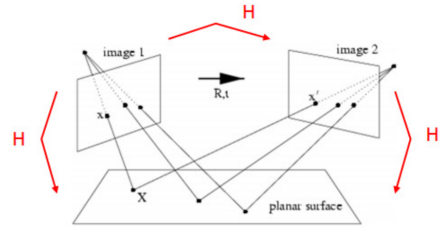

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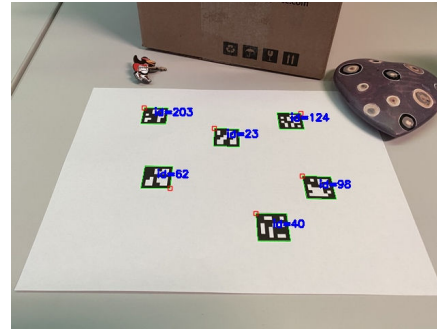
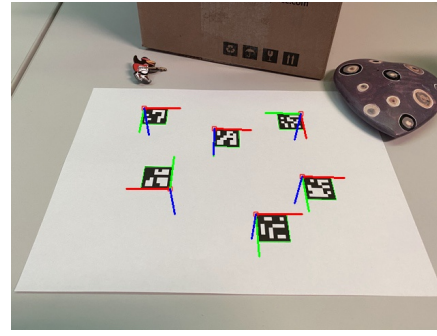
- Pose estimation
  - Estimates camera-object pose
  - Uses PnP algorithm
  - Requires K and correspondences





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  3. Use an Aruco marker grasped by the robot
     - Use FK to compute position of the marker in the robot frame
     - We need to move in the plane of interest
     - We will not have precise estimation of marker w.r.t. hand
- Limited to plane-to-plane mapping

Calibrating camera pose

- Hand-eye calibration
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  - $X = T_{\mathsf{TR}}$
  - $Y = T_{\mathsf{CG}}$

Code for $AX = YB$ calibration

```
def solve_AX_YB(a: list[SE3], b: list[SE3]) -> tuple[SE3, SE3]:
    """Solve A^iX=YB^i, return X, Y"""

    rvec_a = [T.rotation.log() for T in a]
    tvec_a = [T.translation for T in a]
    rvec_b = [T.rotation.log() for T in b]
    tvec_b = [T.translation for T in b]

    Rx, tx, Ry, ty = cv2.calibrateRobotWorldHandEye(rvec_a, tvec_a, rvec_b, tvec_b)
    return SE3(tx[:, 0], SO3(Rx)), SE3(ty[:, 0], SO3(Ry))
```

Influence of Aruco marker size

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- Solution: minimization of reprojection errors
  - Original measurements are in pixels
  - We can perform fine optimization in pixel space
  - Reprojection error:
    - $X^*, Y^* = \arg\min_{X,Y}(\sum_i \boldsymbol{u}_i - \pi(\boldsymbol{x}_i(X, Y)))$
    - $\boldsymbol{u}_i$ - detected 2D point, $\boldsymbol{x}_i$ - 3D point in camera frame, π - projection
    - $\boldsymbol{x}_i$ is computed by FK and X and Y poses
    - Initial guess is from Hand-eye calibration

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  - FK is adjusted to include joint offsets
- Calibration is not a one-time task
  - Calibrate robot before each important task
  - Calibrate robot after any significant change

Conclusion

- Calibration depends on the task
  - Homography
  - Camera intrinsics
  - Camera pose
  - Robot kinematics
- Calibration is not a one-time task
  - Good to automate it

Laboratory

- Laboratory at KN
- Hoop detection and homography