



Robotics: Denavit–Hartenberg notation

Vladimír Petřík

vladimir.petrík@cvut.cz

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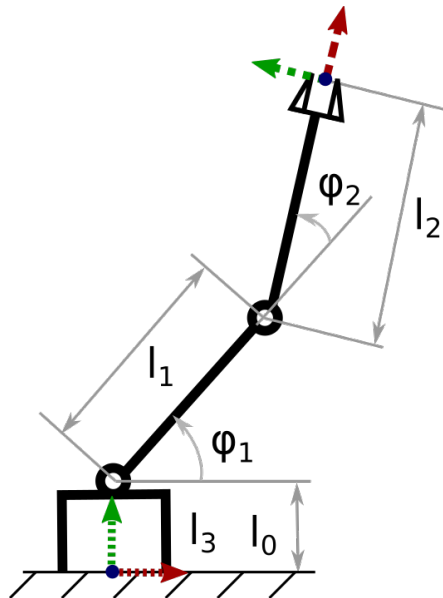
Denavit–Hartenberg notation

- ▶ Method for assigning frames to links in kinematic chains
- ▶ Introduced by Jacques Denavit and Richard Hartenberg in 1955
- ▶ Minimal representation
- ▶ Sometimes used in robotics



Motivation

- ▶ Consider FK for a planar 2-DoF manipulator
 $\varphi_1, \varphi_2 \rightarrow T \in SE(2)$
- ▶ $T_1 = T_y(l_0)$
- ▶ $T_2 = R(\varphi_1)T_x(l_1) \leftarrow \text{structure}$
- ▶ $T_3 = R(\varphi_2)T_x(l_2)$
- ▶ $T = T_1T_2T_3$



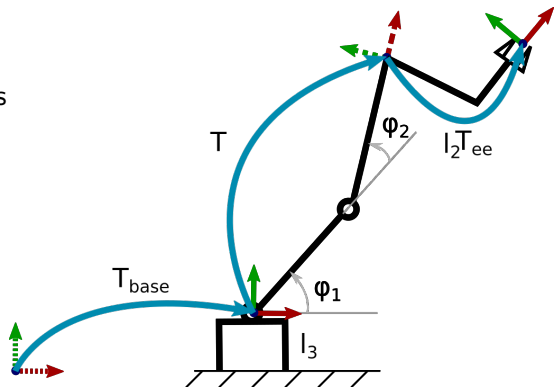
Denavit–Hartenberg parameters

- ▶ Similar structure but for spatial manipulators
- ▶ Four parameters for each transformation
 - ▶ $T_x(a), T_z(d), R_x(\alpha), R_z(\theta)$
 - ▶ $T_{DH} = R_z(\theta)T_z(d)R_x(\alpha)T_x(a)$
- ▶ Which of the following equals to T_{DH} ?
 1. $T_{DH} = T_z(d)R_z(\theta)T_x(a)R_x(\alpha)$
yes, $T_xR_x = R_xT_x$
 2. $T_{DH} = R_x(\alpha)T_x(a)R_z(\theta)T_z(d)$
- ▶ Can we create arbitrary $SE(3)$ transformation with DH
 - ▶ No, only 4 DoF
 - ▶ Designed for open kinematic chains with revolute and prismatic joints
- ▶ Coordinate frames need to be placed appropriately
 - ▶ z -axis is in axis of rotation/translation
 - ▶ x_1 is perpendicular to z_0 and z_1
 - ▶ x_1 intersects z_0 and z_1



Initial and final transforms

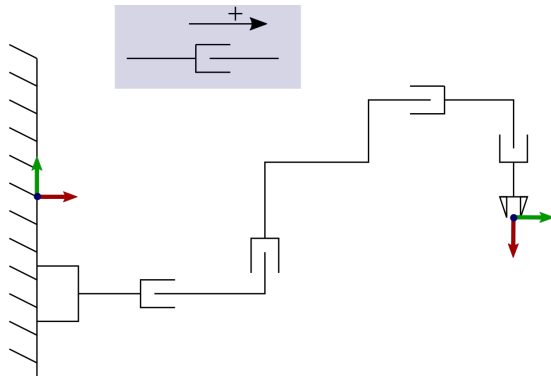
- ▶ We cannot create arbitrary $SE(3)$ transformation with DH
 - ▶ Mount gripper on different location
 - ▶ Defining different reference frame
- ▶ Usually we define initial and final transforms
 - ▶ $T = T_{DH}^1 T_{DH}^2 \dots T_{DH}^n$
 - ▶ $T_{FK} = T_{base} T T_{ee}$



Example: spatial robot in plane

$$xyz = rgb$$

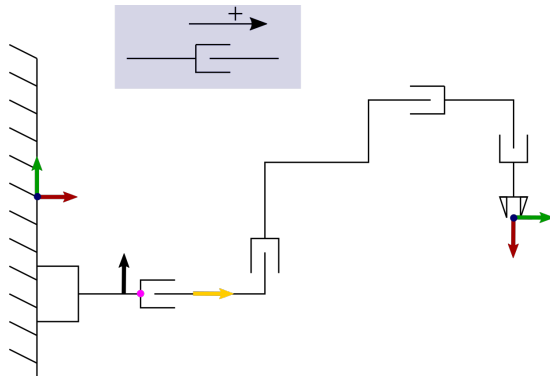
- ▶ Four prismatic joints
- ▶ Solve FK with DH notation



Example: spatial robot in plane

$$xyz = rgb$$

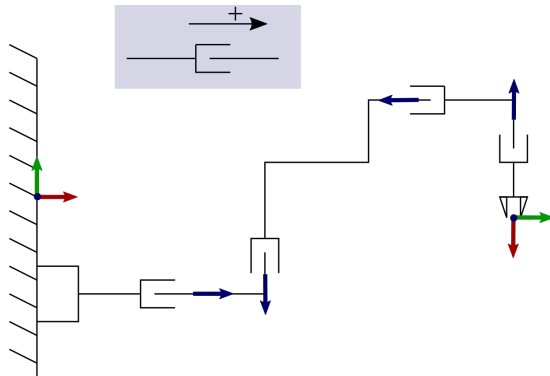
- ▶ z -axis is in axis of rotation/translation
- ▶ Where will be z -axis?
 1. black
 2. yellow
 3. pink



Example: spatial robot in plane

$$xyz = rgb$$

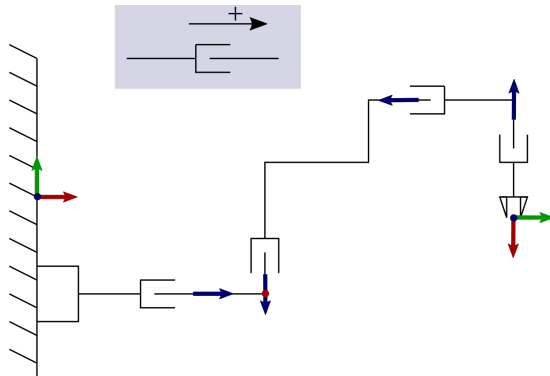
- Be careful with orientation



Example: spatial robot in plane

$$xyz = rgb$$

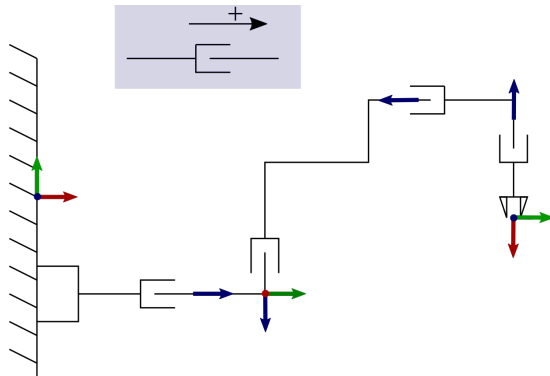
- ▶ We know:
 - ▶ x_1 is perpendicular to z_0 and z_1
 - ▶ x_1 intersects z_0 and z_1
- ▶ x -axis of the first frame:
 1. axis points out of the screen
 2. axis points into the screen
 3. both in/out is correct

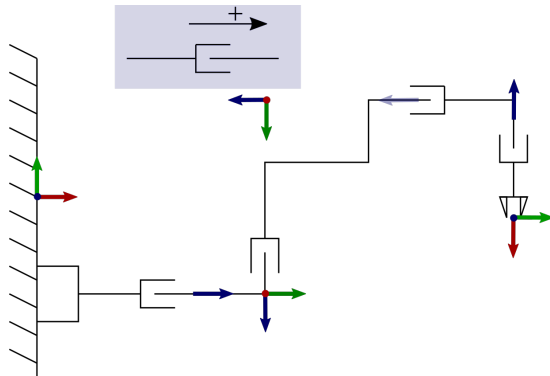


Example: spatial robot in plane

$$xyz = rgb$$

- We know x and z ,
we can determine origin and y

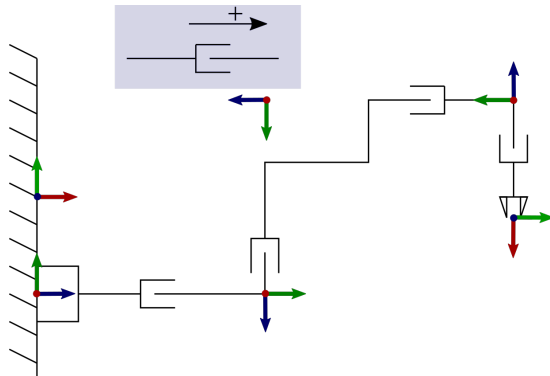




Example: spatial robot in plane

$$xyz = rgb$$

- ▶ We have 6 frames
 - ▶ Initial transformation
 - ▶ 4 DH transformations
 - ▶ Final transformation
- ▶ It remains to determine DH parameters



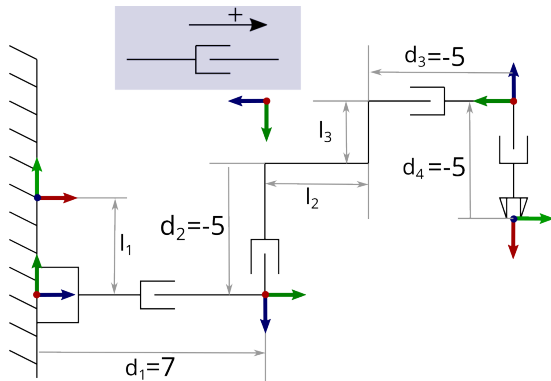
Example: spatial robot in plane

xyz=rgb

- Initial transformation: $T_y(-l_1)R_y(90^\circ)$

JointType	θ	d	a	α
P	0	d_1	0	90°
P	0	$d_2 - l_3$	0	90°
P	0	$d_3 - l_2$	0	90°
P	0	d_4	0	0

- We need to include helper frame before the gripper
 - x_1 is not perpendicular to z_0 and z_1
- Final transformation: $R_y(90^\circ)R_x(180^\circ)$



Conclusion

- ▶ What is DH notation
 - ▶ $T_{\text{DH}} = R_z(\theta)T_z(d)R_x(\alpha)T_x(a)$
 - ▶ Designed for open kinematic chains with revolute and prismatic joints
- ▶ How to assign frames

