Robotics: Path and trajectory generation

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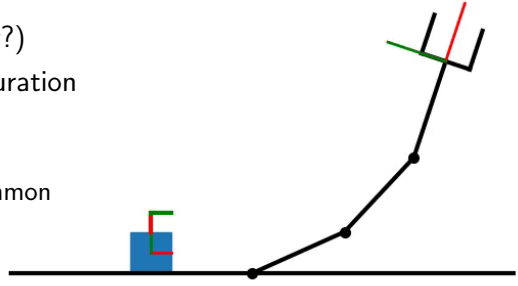
Motivation: pick a cube

- Detect where the cube is in $SE(2)$, $SE(3)$
- Define handle(s) w.r.t. cube
- Compute gripper pose
- Solve IK (select one of the solutions, how?)
- Send robot to selected joint-space configuration
- What motion will robot follow?
  - depends on the robot
  - linear interpolation in joint space is common
  - what is motion?

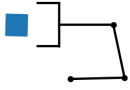


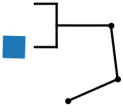
Motion

- Path
  - Geometrical description (sequence of configurations)
  - No timestamps, dynamics, or control restrictions
  - $\boldsymbol{q}(s) \in \mathcal{C}_{\text{free}}, s \in [0, 1]$
  - Main assumption is that trajectory can be computed by postprocessing
- Trajectory
  - Robot configuration in time
  - $\boldsymbol{q}(t) \in \mathcal{C}_{\text{free}}, t \in [0, T]$

Grasping path

- Let us focus on **path** first
- Is grasping path safe? Depends on the start configuration.

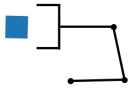


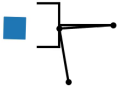


Pre-grasp pose

- We can define pre-grasp pose
  - *e.g.* 5 cm away from the object, w.r.t. handle
  - how to define 5 cm away? By design of handle.
  - fix handle orientation to have x-axis pointing towards the object
  - gripper orientation to have x-axis pointing out of gripper
  - grasp pose T_{RH}
  - if gripper T_{RG} equals T_{RH}, object is grasped
  - pre-grasp pose $T_{RP} = T_{RH} T_x(-\delta_{\text{pre\_grasp}})$
- Is path from pre-grasp to grasp safe if $\delta_{\text{pre\_grasp}}$ is small?
- Is path from pre-grasp to grasp safe if $\delta_{\text{pre\_grasp}}$ is large?

Pre-grasp pose

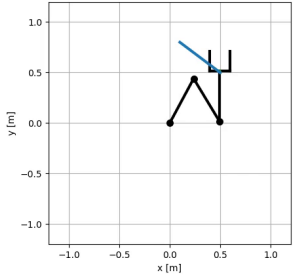


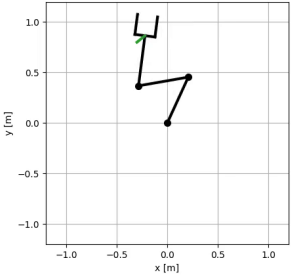


Interpolation in joint space

- Also called straight-line path, point-to-point path
- Start $\boldsymbol{q}_{\text{start}}$
- Goal $\boldsymbol{q}_{\text{goal}}$
- $\boldsymbol{q}(s) = \boldsymbol{q}_{\text{start}} + s(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}}), \quad s \in [0, 1]$
- Easy to compute, well defined
- What is the motion of the gripper?
  - likely not straight-line (for revolute joints)
  - combinations of circular paths (for revolute joints)

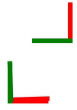
Interpolation in joint space

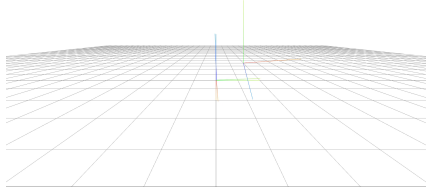




Interpolation in $SE(2)$ and $SE(3)$

- Straight-line path in task space
  - position $\boldsymbol{t}(s) = \boldsymbol{t}_{\text{start}} + s(\boldsymbol{t}_{\text{goal}} - \boldsymbol{t}_{\text{start}}), \quad s \in [0, 1]$
  - rotation $R(s) = R_{\text{start}} \exp\left(s \log(R_{\text{start}}^{-1} R_{\text{goal}})\right), \quad s \in [0, 1]$



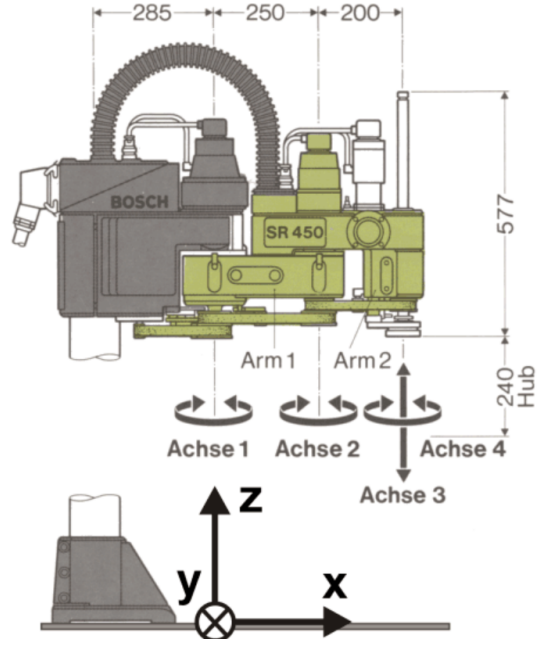


Joint-space path from task-space path

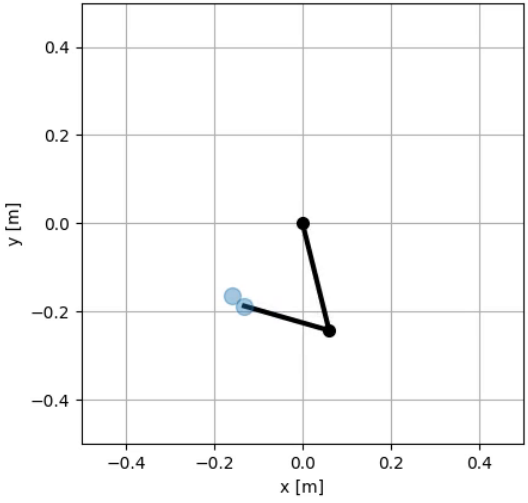
- Compute $\boldsymbol{q}(s)$ from $T_{RG}(s)$
- Solve IK for each s and pick the first solution of IK?
  - we did not define what is *first* solution of IK
  - let us use the closest solution of IK
  - can it happen that closest solution is not *close enough*? **yes**, let us see an example

SCARA robot

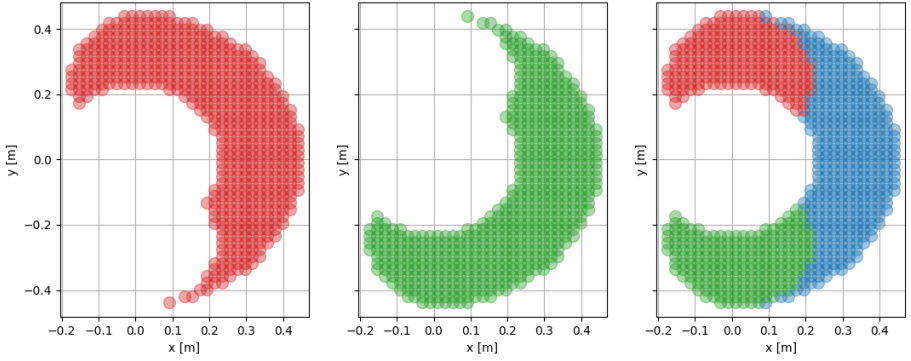
-
- Analyze kinematics of SCARA
- Structure RRPR
- Self-collisions avoided by joint limits
  - $\pm 85°$
  - $\pm 120°$
  - $(-330\ \text{mm}, 5\ \text{mm})$
  - $(-20°, 1080°)$
- Compute FK and IK in xy-plane



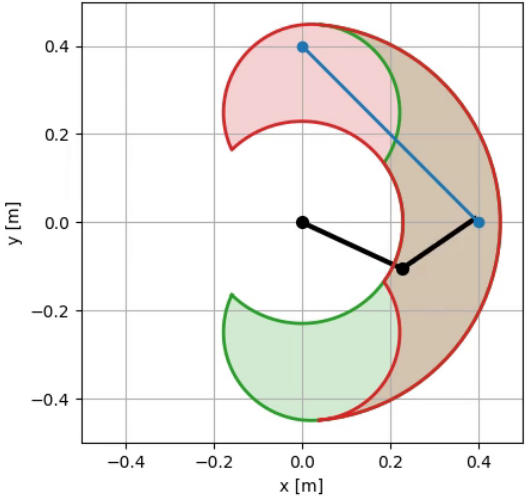
SCARA robot workspace



SCARA robot IK



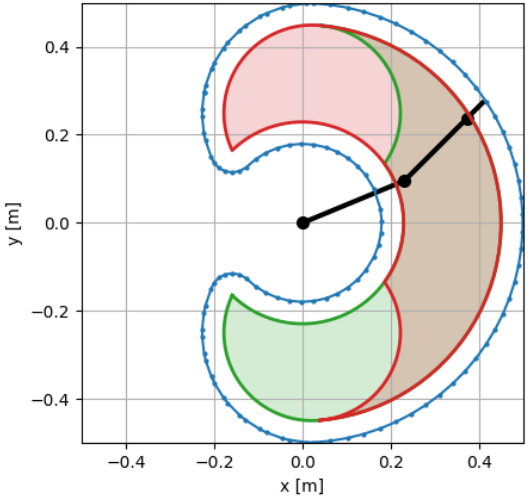
Task-space interpolation



Task space interpolation

- Not all solutions of IK are available everywhere
- We need to resolve jumps in configuration space
- To change the configuration we need to pass via singularity
- The task-space interpolation can be used for pre-grasp to grasp path

SCARA effect of the last link



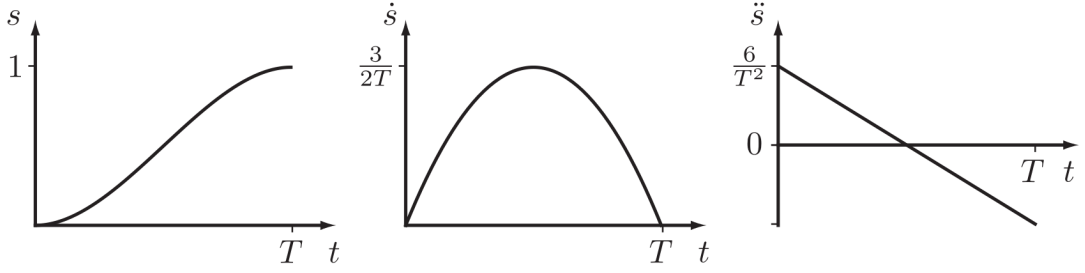
Trajectory from path

- Time scaling $s(t)$, $t \in [0, T]$, $s : [0, T] \to [0, 1]$
- A path and time scaling defines trajectory $\boldsymbol{q}(s(t))$
- Derivations:
  - velocity: $\dot{\boldsymbol{q}} = \frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}s}\dot{s}$
  - acceleration: $\ddot{\boldsymbol{q}} = \frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}s}\ddot{s} + \frac{\mathrm{d}^2\boldsymbol{q}}{\mathrm{d}s^2}\dot{s}$

Straight-line path time scaling

- Path
  - position: $\boldsymbol{q}(s) = \boldsymbol{q}_{\text{start}} + s(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}}), \quad s \in [0, 1]$
  - velocity: $\dot{\boldsymbol{q}} = \dot{s}(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}})$
  - acceleration: $\ddot{\boldsymbol{q}} = \ddot{s}(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}})$
- 3rd order polynomial time scaling
  - $s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$
  - $\dot{s}(t) = a_1 + 2a_2 t + 3a_3 t^2$
  - constraints: $s(0) = \dot{s}(0) = 0$, $s(T) = 1$, $\dot{s}(T) = 0$
  - solution that satisfies constraints: $a_0 = 0, \quad a_1 = 0, \quad a_2 = 3/T^2, \quad a_3 = -2/T^3$
- Trajectory
  - $\boldsymbol{q}(t) = \boldsymbol{q}_{\text{start}} + \left(\frac{3t^2}{T^2} - \frac{2t^3}{T^3}\right)(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}})$
  - $\dot{\boldsymbol{q}}(t) = \left(\frac{6t}{T^2} - \frac{2t^2}{T^3}\right)(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}})$
  - $\ddot{\boldsymbol{q}}(t) = \left(\frac{6}{T^2} - \frac{12t}{T^3}\right)(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}})$

3rd order polynomial time scaling

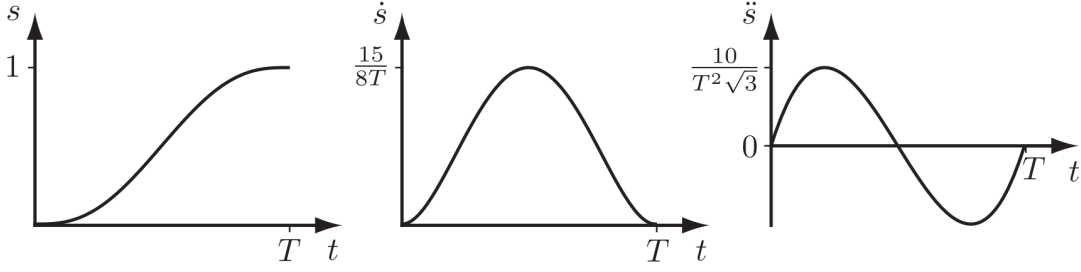


Straight-line path time scaling

- Maximum joint velocities:
  - $t = T/2$
  - $\dot{\boldsymbol{q}}_{\text{max}} = \frac{3}{2T}(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}})$
- Maximum joint acceleration:
  - $t = 0$ and $t = T$
  - $\ddot{\boldsymbol{q}}_{\text{max}} = \left\| \frac{6}{T^2}(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}}) \right\|$
  - $\ddot{\boldsymbol{q}}_{\text{min}} = -\left\| \frac{6}{T^2}(\boldsymbol{q}_{\text{goal}} - \boldsymbol{q}_{\text{start}}) \right\|$
- How to use this information?
  - check if requested motion T is feasible given the velocity/acceleration limits
  - find minimum T such that velocity and acceleration constraints are satisfied

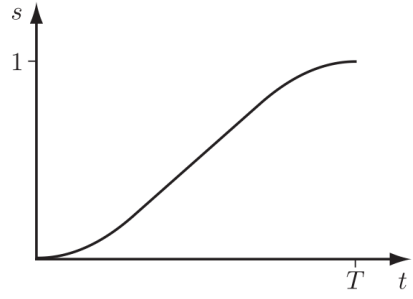
5th order polynomial

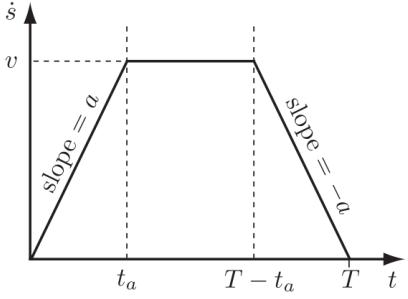
- 3rd order polynomial does not enforce zero acceleration at the beginning and end
  - infinite jerk (derivative of acceleration)
  - can cause vibrations
- We can use 5th order polynomial



Trapezoidal time scaling

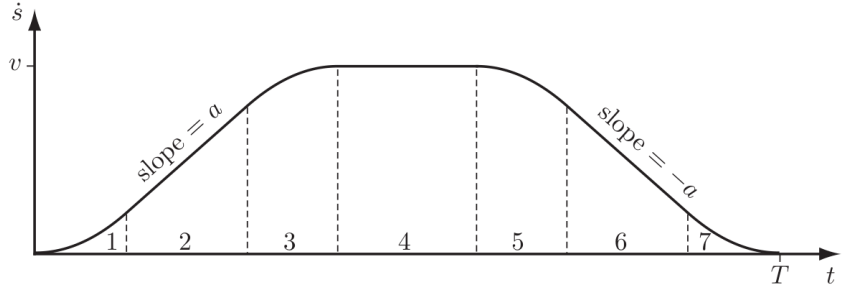
- Constant acceleration phase
- Constant velocity phase
- Constant deceleration phase
- Not smooth but it is the fastest straight-line motion possible





S-Curve time scaling

- Trapezoidal motions cause discontinuous jumps in acceleration
- S-curve smooths it to avoid vibrations
  - constant jerk, constant acceleration, constant jerk, constant velocity, constant jerk, constant deceleration, constant jerk

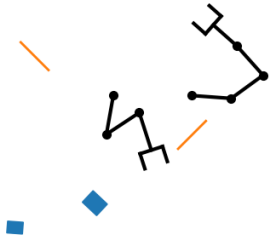


TAMP: Task and Motion Planning

Motivation

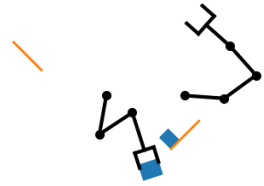
- We know how to plan motion for a robot in robot's configuration space
  - manually define handle on object
  - computer grasp and pre-grasp for detected object's pose
  - plan motion to pre-grasp
  - interpolate to grasp, grasp
  - interpolate to pre-grasp
  - plan motion to pre-place, place, release, pre-place
- What if we have many handles? Many objects?
- **Manipulation Task and Motion Planning (TAMP)**
  - simultaneously plan task and motion solutions
  - task is the sequence of grasps and placements (discrete space)
  - motion is the sequence of robot configurations (continuous space)
  - Humanoid Path Planning (HPP) software approach

Configuration Space



- Multiple grippers connected to robots
- Environment surfaces that can be used for placing an object
- Multiple objects
  - multiple handles per object
  - multiple contact surfaces per object
- Configuration space is the set of all possible configurations of all objects and robots
  - $\mathcal{C} = \mathbb{R}^{N_1} \times \mathbb{R}^{N_2} \ldots \times SE(3)^M$
  - N_i DoF of the i-th robot
  - M number of objects
  - however, not all configuration are feasible
  - constraints are used to define feasible configurations

Constraints

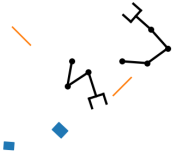


- Object is placed or grasped, i.e. cannot fly
- Placement constraint
  - object lies on a surface
  - numerical constraints
  - **object surface** is placed on an **environment surface**
- Grasp constraint
  - object is grasped by a gripper
  - numerical constraint
  - **handle frame** equals **gripper frame**

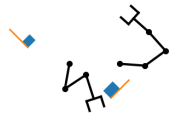
Stav (state)

- State is a set of constraints
- Manifold of feasible configurations in the configuration space
- For example, one state can be defined by constraining both objects
  - object O_1 is placed on the surface E_1 via object surface S_1
  - object O_2 is grasped by the gripper G_1 via handle H_1
- How to sample configuration from a state?
  - sample from the $\mathcal{C}$
  - geometric projection to satisfy all the constraints
  - numerical optimization (Newton-Raphson) to satisfy all the constraints

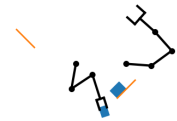
Sampling from states



Sample from $\mathcal{C}$



- Project to state:
  - O_1 placed on E_1 via S_1
  - O_2 placed on E_2 via S_1



- Project to state:
  - O_1 grasped by G_1 via H_1
  - O_2 placed on E_2 via S_1

Transitions

- Transition defines motion between two states
  - identity transition allows to move robot inside the state
  - place transition allows to move object from the gripper to the surface
  - grasp transition allows to move object from the surface to the gripper
- Sampling on transitions vs sampling on states
  - transition respect constraints from the given state
  - for example, identity on place state will not move object (sampling on state can move object)
  - grasp transition is specified to move via pre-grasp
  - place transition is specified to move via pre-place

Interpolation on transition

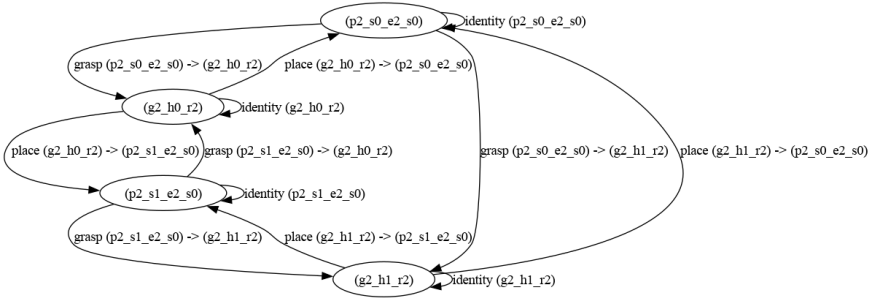
- Interpolate between two configurations but respect constraints of the states/transition

Grasp

Place

Constraint graph

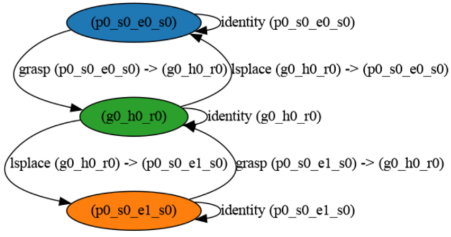
- Defines all possible transitions between existing states
- Example: single arm, one object

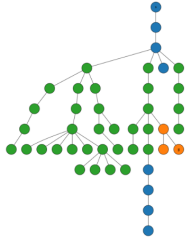


RRT on constraint graph

- Random sampling q_{rand}
  - sample random transition
  - select random existing configuration from the transition source
  - sample random configuration from the transition target reachable from beginning
- Nearest neighbor q_{tree}
  - node that is closest to q_{rand} via interpolation on the transition
- Local planner uses interpolation on transition

RRT on constraint graph

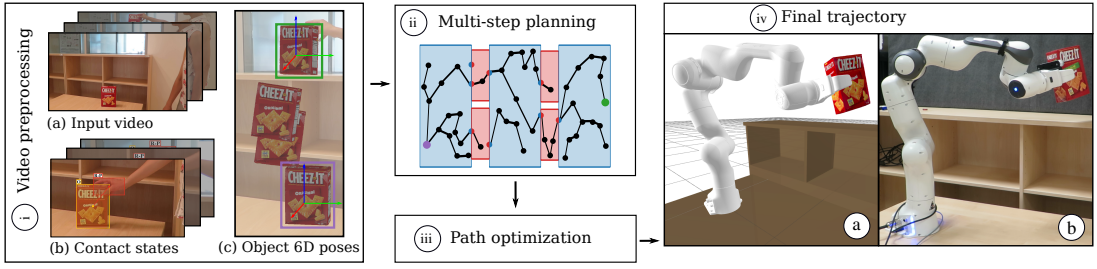




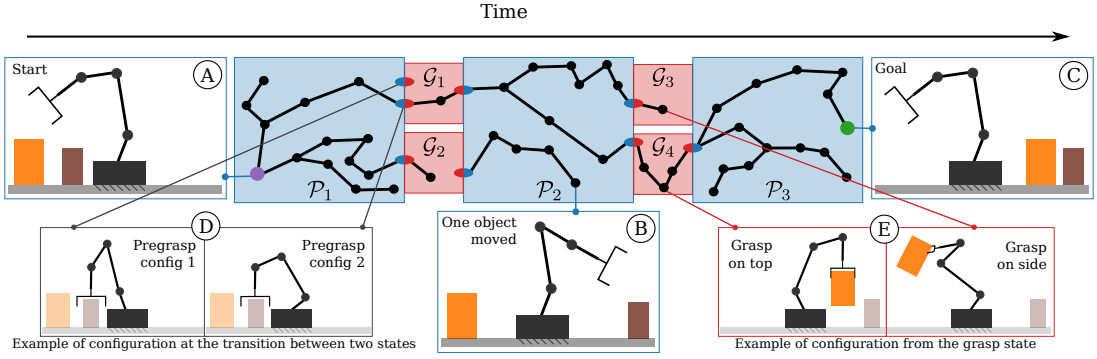
Conclusion

- Configuration space for TAMP is complex
  - discrete set of states
  - continuous motion
  - encoded by constraint graph that allow us to use RRT
- Usually not used in industry
  - task space sequence is hard-coded by programmers
  - only motion is found by motion planners (if cannot be hard-coded)
- How to avoid hard-coding? Video demonstration.

TAMP guided by video



TAMP guided by video



Summary

- Path/Trajectory
- Grasping path generation
- Interpolation in joint space and task space
- Time scaling parameterization
- TAMP

Laboratory (KN)

- Poslední oraganizované cvičení
- Implementace PRM
- Domácí úkol: implementace RRT a Random shortcut [dobrovolné]