Robotics: Closed kinematics chains

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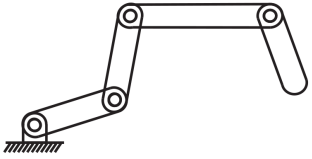
Open/Closed kinematic chain

- Open kinematics chains: no loops
- Closed kinematic chains contains loops



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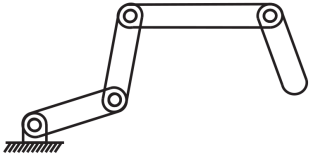
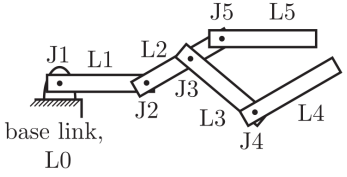


Open - sequential



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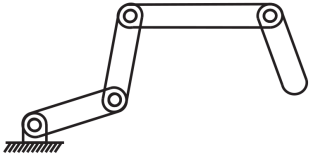

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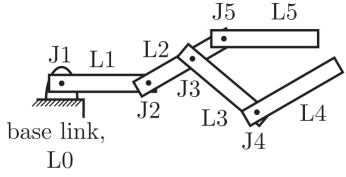
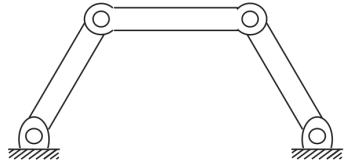
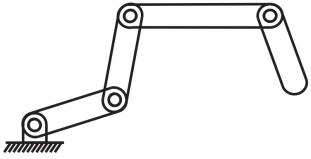


Open - tree



Open/Closed kinematic chain

- Open kinematics chains: no loops
- Closed kinematic chains contains loops
- Many closed kinematics chains can be expressed with open kinematics chains



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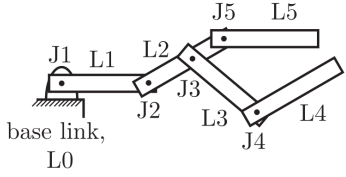
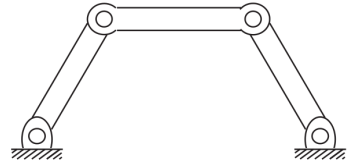


Open - tree



Closed

Examples of closed kinematics chain

https://www.youtube.com/watch?v=v1x64Zg1-hE

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https://www.youtube.com/watch?v=mWmbI99IRq8

Examples of closed kinematics chain

https://www.youtube.com/watch?v=B756Novxejo

Examples of closed kinematics chain

https://www.youtube.com/watch?v=WmKnnp1xTPg&list=PLXD3wdKvVE38twjOHvSbQP7UeR0MLLqVp

Closed kinematics chains

- Closed kinematics chains
  - Contain loops
  - Are more difficult to control
  - Typically small workspace
  - Are more difficult to analyze
- Advantages
  - Can be redundantly actuated
  - Mechanical advantage: faster, stronger, or stiffer
- Grübler's formula can be used to determine number of DoF

Grübler's formula

- $n_{\text{DoF}} = m\,(L-1) - \sum_{i=1}^{N} c_i$
- L is number of links including ground
- N is number joints
- m is DoF of rigid body (3 for planar, 6 for spatial)
- c_i number of constrains provided by joint i

Grübler's formula

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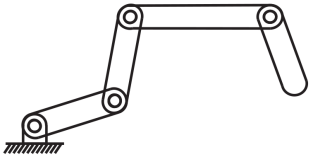
- f_i number of freedoms provided by joint i
- $f_i + c_i = m$
- Works for *generic* cases, fails under certain configurations - when joints constrains are not independent

Applications of Grübler's formula

$$n_{\text{DoF}} = m\,(L - 1 - N) + \sum_{i=1}^{N} f_i$$

m - body DoF, L - number of links, N - number of joints, f_i - joint DoF



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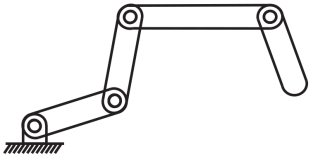

$3(5-1-4)+(1+1+1+1) =$
4 DoF

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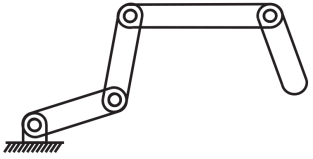
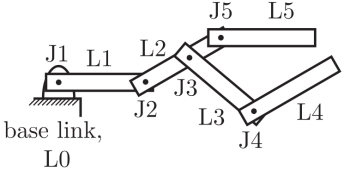


$3(5-1-4)+(1+1+1+1) = 4$ DoF

$3(6-1-5)+(1+1+1+1+0) = 4$ DoF (if fixed joint)



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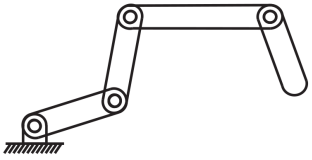

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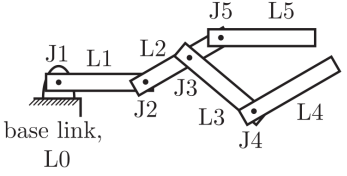
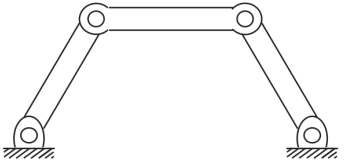


$3(6-1-5)+(1+1+1+1+1) = 5$ DoF



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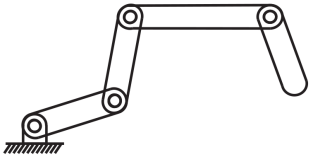

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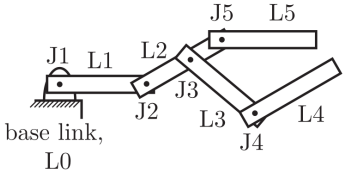

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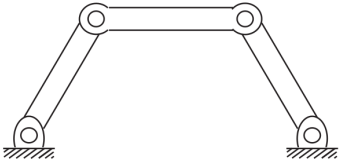

$3(4-1-4)+(1+1+1+1) = 1$ DoF

Applications of Grübler's formula

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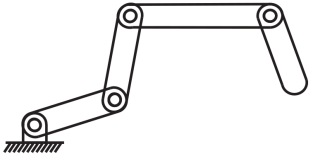

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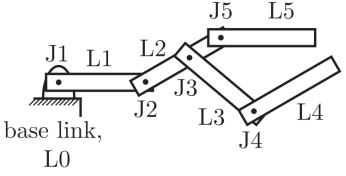

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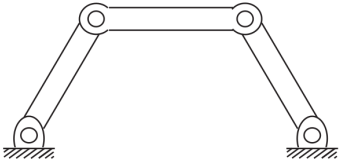

$3(4-1-4)+(1+1+1+1) = 1$ DoF

$3(5-1-5)+(1+1+1+1+0) = 1$ DoF (if two grounds connected by fixed joint)

Examples of kinematics chains

- Open kinematics chains
  - Structure RR, Task space $\mathbb{R}^2$

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- Closed kinematics chains
  - Structure RRRR, Task space $\mathbb{R}^1$

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  - Structure RRRRRR, Task space $SE(2)$
  - Structure RRRP, Task space $\mathbb{R}^1$

Spatial inverse kinematics for Mitsubishi RV6S